Reducing Radicals

To reduce radical expressions, you must use prime factorization and apply the following properties:

,

The first of these properties is the most important for reducing radicals, but we will use the others (especially the second one) when simplifying.

Let’s look at our first example.

Simplify: Start by finding the prime factorization. 2|200

2|100

2| 50

5| 25

5

So, the prime factorization is:

Now, rewrite the problem.

Now, apply the 2nd property from above.

The 23 can be separated into 2 and 22.

Apply the 1st property from above and simplify.

You may have noticed a slight difference between what we had when we applied the 1st property and what was written in the property, so let us look at the difference.

As written above: What we had:

What’s the difference? In the example at the top of the page, the exponent was applied to the whole radical expression, whereas, in our problem, the exponent was only on the radicand. In many instances in math, this is a very important distinction. But, as it turns out, they end up being the same thing in this case. Let’s look at why.

So, effectively, they are the same thing.

Let’s look at another example, using an index greater than 2.

Simplify: Start by finding the prime factorization. 2|432

2|216

2|108

2| 54

3| 27

3| 9

3

So, the prime factorization is:

Now, rewrite the problem.

Now, apply the 2nd property from above.

The 24 can be separated into 2 and 23.

Apply the 1st property from above and simplify.

Notice that, in both of the above examples, we needed exponents that were equal to the index in order to apply the 1st property the way that we did. The bases to the power of 1 (no exponent showing) must remain in the radical, while larger exponents can be broken down into smaller ones.

If, for instance, we had an exponent of 2 on a base under a radical with an index of 4, we would not be able to simply any further.

Here are some practice problems. Their answers are on the next page.

Simplify if possible:

Answers to practice:

(1) (2) (3) (4) (5) (Cannot be reduced.)

How we got there:

(1) Prime Factorization: 2|756

2|378

3|189

3| 63

3| 21

7

(2) Prime Factorization: 3|147

7| 49

7

(3) Prime Factorization: 5|3125

5| 625

5| 125

5| 25

5

(4) Prime Factorization: 2|392

2|196

2| 98

7| 49

7

(5) Prime Factorization: 2|60 There are no factors with a

2|30 power of 3 (the index), so

3|15 this cannot be further simplified.

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